**Discrete Mathematical Structures**

**Week-1**

**Long Descriptive Questions**

**1, Which of the following is an equivalence relation?**

**a) The relation R on Z defined by aRb if a 2 − b 2 ≤ 7.**

**b) The relation R on Z defined by aRb if 2a + 5b ≡ 0 (mod 7).**

**c) The relation R on Z defined by aRb if a + b ≡ 0 (mod 5).**

**d) The relation R on Z defined by aRb if a 2 + b 2 = 0**

To be an equivalence relation, a relation on a set must satisfy three properties:

**Reflexive**: For all a in the set, aRa.

**Symmetric**: For all a, b in the set, if aRb, then bRa.

**Transitive**: For all a, b, c in the set, if aRb and bRc, then aRc.

• The relation R on Z defined by aRb if a^2 - b^2 ≤ 7. This relation is not reflexive because a^2 - a^2 = 0, which is not less than or equal to 7 for all integers a. Therefore, it fails the reflexivity property and is not an equivalence relation

• The relation R on Z defined by aRb if 2a + 5b = 0 (mod 7) is not an equivalence relation because it is not transitive. For example, let a = 1, b = 5, and c = 2. Then, aRb because 2(1) + 5(5) = 27 ≡ 0 (mod 7) and bRc because 2(5) + 5(2) = 20 ≡ 0 (mod 7). However, aRc is false because 2(1) + 5(2) = 12 ≢ 0 (mod 7)

• The relation R on Z defined by aRb if a + b = 0 (mod 5) is an equivalence relation because it satisfies all three properties.

**Reflexive**: For any integer a, a + a = 2a ≡ 0 (mod 5), so aRa.

**Symmetric:** If aRb, then a + b ≡ 0 (mod 5), which implies b + a ≡ 0 (mod 5), so bRa.

**Transitive:** If aRb and bRc, then a + b ≡ 0 (mod 5) and b + c ≡ 0 (mod 5), which implies a + c = (a + b) + (b + c) ≡ 0 + 0 = 0 (mod 5), so aRc

• The relation R on Z defined by aRb if a^2 + b^2 = 0 is an equivalence relation because it satisfies all three properties.

**Reflexive:** For any integer a, a^2 + a^2 = 2a^2 ≠ 0 unless a = 0, so aRa holds only for a = 0.

**Symmetric:** If aRb, then a^2 + b^2 = 0, which implies b^2 + a^2 = 0, so bRa.

**Transitive:** If aRb and bRc, then a^2 + b^2 = b^2 + c^2 = 0, which implies a^2 + c^2 = (a^2 + b^2) + (b^2 + c^2) = 0 + 0 = 0, so aRc.

Therefore, the equivalence relations are (c) and (d)

**2, Consider the set S = R, a real number where x ∼ y if and only if x 2 = y 2 . Prove that "∼" is equivalence relation**

**Reflexivity:** For all x ∈ S, x ∼ x.

**Symmetry:** For all x, y ∈ S, if x ∼ y, then y ∼ x.

**Transitivity:** For all x, y, and z ∈ S, if x ∼ y and y ∼ z, then x ∼ z.

Let's prove each of these properties for the relation ∼ defined on S = R (the set of real numbers) where x ∼ y if and only if x² = y².

**Reflexivity:**

For all x ∈ R, x ∼ x if and only if x² = x², which is always true for any real number. Therefore, ∼ is reflexive.

**Symmetry:**

For all x, y ∈ R, if x ∼ y, then x² = y². Since equality is symmetric, if x² = y², then y² = x². This implies that y ∼ x. Therefore, ∼ is symmetric.

**Transitivity:** For all x, y, and z ∈ R, if x ∼ y and y ∼ z, then x² = y² and y² = z². By transitivity of equality, we can conclude that x² = z². Therefore, x ∼ z. Thus, ∼ is transitive.

Since ∼ is reflexive, symmetric, and transitive, it satisfies all three properties of an equivalence relation. Therefore, ∼ is indeed an equivalence relation

**3, Let A = {1, 2, 3, 4} and B = {a, b, c, d, e}. Define f = {(1, b),(2, c),(3, a),(4, e)}. Choose the most complete correct statement below.**

a) f is an equivalence relation,

b) f is a surjective function from A to B.

c) f is a function from A to B,

d) f is a bijective function from A to B.

e) f is an injective function from A to B,

f ) All of the above.

g) None of the above

**c) f is a function from A to B:**

This statement is correct. f is indeed a function from A to B because for each element in A, there is a unique corresponding element in B in the relation f

**4, Which of the following is an equivalence relation?**

**a) The relation R on Z defined by aRb if a 6 − b 6 ≤ 7.**

**b) The relation R on Z defined by aRb if 5a + 8b ≡ 0 (mod 7).**

**c) The relation R on Z defined by aRb if 5a + 6b ≡ 0 (mod 5).**

To check if a relation is an equivalence relation, we need to verify three properties: reflexivity, symmetry, and transitivity.

a) **The relation R on Z defined by aRb if a ≤ 7 and b ≤ 7** is not an equivalence relation because it is not transitive. For example, if a = 5, b = 6, and c = 7, then aRb and bRc, but not aRc.

b) **The relation R on Z defined by aRb if 5a + 8b = 0 (mod 7)** is an equivalence relation.

**Reflexivity:** For any a ∈ Z, 5a + 8a = 13a ≡ 0 (mod 7), so aRa.

**Symmetry:** If aRb, then 5a + 8b ≡ 0 (mod 7), which implies that 5b + 8a ≡ 0 (mod 7), so bRa.

**Transitivity:** If aRb and bRc, then 5a + 8b ≡ 0 (mod 7) and 5b + 8c ≡ 0 (mod 7), which implies that 25a + 40b ≡ 0 (mod 7) and -40b - 64c ≡ 0 (mod 7). Adding these two congruences, we get 25a - 64c ≡ 0 (mod 7), which implies that aRc.

c) The relation R on Z defined by aRb if 5a + 6b ≡ 0 (mod 5) is not an equivalence relation because it is not transitive. For example, if a = 1, b = 4, and c = 2, then aRb and bRc, but not aRc